A comparison of critical state behaviour between triaxial and simple shear conditions: A DEM study

Une comparaison du comportement à l'état critique entre les conditions de cisaillement triaxiales et simples: une étude DEM

Md. Mizanur Rahman & Hoang Bao Khoi Nguyen
UniSA STEM, University of South Australia, SA 5000, Adelaide, South Australia, Australia

ABSTRACT: The critical state (CS) theory has been well evaluated via triaxial testing. The shearing condition in such test considers the inequalities in the principal stresses and strains. However, such shearing condition may not be always the case in the real-life scenario, a pure shear phenomenon in which the strain or stress is applied in the shear direction also occurs. A comparison or relation between these conditions is very limited due to the lack of studies for CS behaviour in both direct simple shear and triaxial conditions for the same soil. Despite this many commonly used geotechnical applications, particularly cyclic liquefaction assessment, require modification of triaxial condition to simple shear condition. Therefore, both triaxial and direct simple shear tests were simulated via a discrete element method (DEM) for the same soil. The study provides in-depth understandings on CS behaviour for these conditions and broadens our knowledge in inferring direct simple shear behaviour from a simpler and widely used triaxial condition.

RÉSUMÉ : La théorie de l'état critique (CS) a été bien évaluée via des tests triaxiaux. La condition de cisaillement dans un tel test prend en compte les inégalités dans les principales contraintes et déformations. Cependant, une telle condition de cisaillement peut ne pas être toujours le cas dans le scénario réel, un phénomène de cisaillement pur dans lequel la déformation ou la contrainte est appliquée dans la direction du cisaillement se produit également. Une comparaison ou une relation entre ces conditions est très limitée en raison du manque d'études sur le comportement des CS à la fois en cisaillement simple direct et en conditions triaxiales pour le même sol. Malgré cela, de nombreuses applications géotechniques couramment utilisées, en particulier l'évaluation de la liquéfaction cyclique, nécessitent la modification de la condition triaxiale en condition de cisaillement simple. Par conséquent, les essais de cisaillement simples triaxiaux et directs ont été simulés via une méthode des éléments discrets (DEM) pour le même sol. L'étude fournit des connaissances approfondies sur le comportement CS pour ces conditions et élargit nos connaissances en inférant le comportement de cisaillement simple direct à partir d'une condition triaxiale plus simple et largement utilisée.

KEYWORDS: Critical state theory; direct simple shear; triaxial, discrete element method; granular material.

1 INTRODUCTION.

The critical state soil mechanics (CSSM) framework for granular materials has been well established in the laboratory experiments (Schofield & Wroth 1968), most of which are undrained and drained triaxial compression tests after isotropic consolidation. With increasing deviatoric strain (ε_q) the soil will eventually reach an equilibrium state, at which there is no change in stresses (deviatoric stress, q or mean effective stress, p'), pore water pressure (Δu) and volume i.e. dq=0, dp'=0, dΔu=0 and dε_v=0, regardless of its initial state. Such equilibrium state is referred to as the critical state (CS) in the triaxial condition, which has been extensively investigated in the classical CSSM framework (Been et al. 1991; Rahman & Lo 2012; Yan & Zhang 2013; Rahman & Lo 2014; Zhang et al. 2018; Wei & Yang 2019). It has been reported that CS is unique for soil and is adopted in many liquefaction investigations in the literature. Note, σ'_{11} and ε_{11} are the major principle effective stress and strain; σ'_{33} (=σ'_{22}) and ε_{33} (=ε_{22}) are the minor principal effective stress and strain; q=(σ'_{11}-σ'_{33}); p'=(σ'_{11}+2σ'_{33})/3; ε_q=2/3(ε_{11}-ε_{33}); ε_v=ε_{11}+2ε_{33}.

The shear in the triaxial condition is the inequalities in the principal stresses or strains, which may not be always the real scenario. Sometimes, a simple shear phenomenon, in which the strain or stress is applied in the real shear direction, can happen. Therefore, the simple shear test has been adopted to study the shearing behaviour of soils (Prevost & Høeg 1976; Atkinson et al. 1991; Yoshimine et al. 1999; Kim 2009; Li et al. 2017). However, the study of CS behaviour in the direct simple shear condition is still limited. Riemer & Seed (1997) reported that CSL of simple shear tests was below CS lines from triaxial compression and extension tests in the e-log(p') space. However, simple shear datapoints were limited and scattered, and other characteristic features were not reported. Differently, Yoshimine et al. (1999) showed the CSL of simple shear tests was sandwiched between the lines from compression and extensive tests. Hence, the criteria to define CS for simple shear condition and the transferring knowledge from triaxial condition to simple shear condition is still not in agreement and yet to be fully explored.

The specimen size, preparation method and load controlling feedbacks for triaxial and simple shear are often different which introduces some uncertainties in a direct comparison of experimental data. Besides, the discrete element method (DEM), which does not have those limitations but allows the observation of soil fabric, has been widely adopted to validate the CS theory (Yang & Dai 2011; Zhao & Guo 2013; Huang et al. 2014; Nguyen et al. 2017). DEM thus enables the development of a qualitative understanding of the response of soil from a micro-mechanics perspective. The CS behaviour and associated micro-mechanical quantities have been extensively investigated in DEM under triaxial condition (Kuhn 2016; Nguyen et al. 2017; Nguyen et al. 2018; Zhao & Zhao 2019). However, the DEM studies for the simple shear condition are still limited. Some DEM studies in simple shear conditions (Bernhardt et al. 2016; Asadzadeh & Soroush 2017; Walker et al. 2017) rather tried to model the behaviour and capture the particle interactions such as force chain or fabric. The analysis of CS behaviour in DEM simple shear simulations has not been truly explored and the effect of shearing condition on CS behaviour in both macro- and micro-mechanics is still missing. Therefore, the main objective of this study is to adopt DEM to validate the CS theory for the simple shear condition and explore the characteristic features of the simple shear condition.

2 MATERIALS AND METHOD

2.1 *DEM specimen preparation*

Cubical assemblies of ellipsoids were generated and then gradually compressed to small isotropic stress. The coefficient of friction at particle contacts (μ) was varied between 0 and 1 during this stage to achieve a desire e_0: a large μ for a higher e_0 and a small μ for lower e_0. Then the assembly was isotropically compressed to a desire p'_0; where subscript "0" denotes the end of isotropic compression. An assembly of 5400 particles was used in this study as shown in Figure 1.

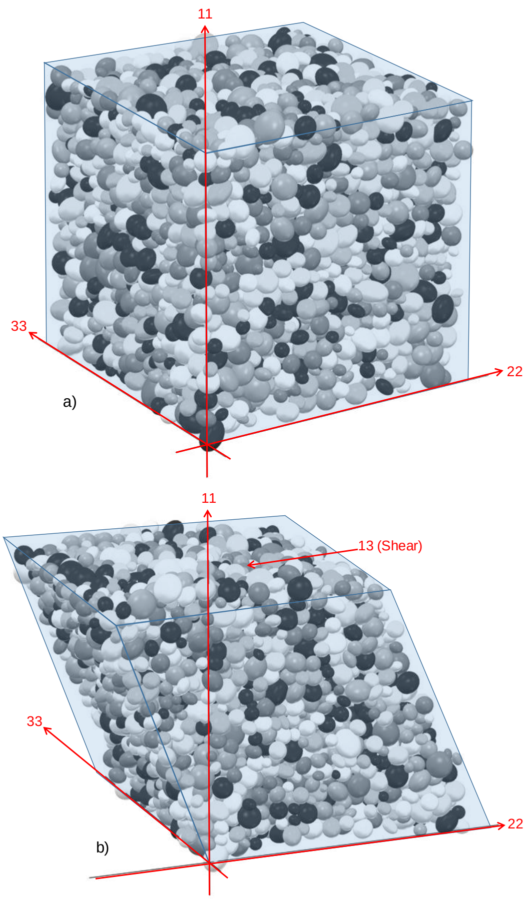

Figure 1. An assembly of 5400 ellipsoids: (a) Assembly before direct simple shear and (b) assembly after direct simple shear.

All particles were assigned a normal contact stiffness of 10^8 N/m, a ratio of tangential to normal contact stiffness of 1.0, a coefficient of friction at particle contacts of 0.50 and the coefficient of (rotational and translational) body damping of 0.05. The detail of input parameters can be found in Table 1.

Table 1. DEM input parameters

Parameters	*Value*	*Unit*
Coefficient of friction	0.5	-
Normal contact stiffness	10^8	N/m
Tangential contact stiffness	10^8	N/m
Coefficient of body damping	0.05	-

2.2 *DEM controlling modes*

2.2.1 *Tensor in DEM*

The DEM simulations were done by controlling the stress ($\boldsymbol{\sigma}'$) and strain ($\boldsymbol{\varepsilon}$) tensors. Each tensor has 9 components, including 3 orthogonal and shear components, as shown in Eq. 1

$$\boldsymbol{\sigma}' = \begin{bmatrix} \sigma'_{11} & \sigma'_{12} & \sigma'_{13} \\ \sigma'_{21} & \sigma'_{22} & \sigma'_{23} \\ \sigma'_{31} & \sigma'_{32} & \sigma'_{33} \end{bmatrix} \quad (1a)$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \quad (1b)$$

2.2.2 *Controlling modes*

All specimens in this study were K_0 consolidated to impose the laboratory condition of simple shear test. In such condition, normal stress ($\sigma'_N=\sigma'_{11}$) is applied and horizontal strains (ε_{33} and ε_{22}) remain constant. Under constant-volume simple shear condition, all orthogonal strain components are kept constant i.e. $d\varepsilon_{11}=d\varepsilon_{22}=d\varepsilon_{33}=0$. Shear is only applied in one direction, say direction 13 i.e. ε_{13}.

3 DEFINING CRITICAL STATE

3.1 *Non-flow behaviour under simple shear condition*

Figure 2 shows that all the stresses i.e. q, p', τ and σ'_N for NF behaviour (SS_CV_02, e_0=0.496, σ'_N=200kPa) increase with increasing shear strain (γ or ε_{13}). They all reach the CS at high strains, which is in line with what has been observed in the triaxial studies. It should be also noted that the σ'_N in Figure 2a has a small reduction in very early strain, which may relate to the so-called phase transformation (PT) state found in the literature of triaxial test. PT state is found at the 'knee' of effective stress path at which dp' or dΔu is zero. In this study, Δu can be measured indirectly based on the difference between drained and undrained stress path in simple shear condition. In Figure 2c, the Δu path initially increases (contractive tendency). After attaining an initial peak Δu, this path gradually decreases (dilative tendency) towards the CS. The initial peak Δu or the transition from contractive to dilative tendency is the PT state in simple condition.

3.2 *Flow behaviour under simple shear condition*

Figure 3 shows SS_CV_06 (e_0=0.609, σ'_N=100kPa) with the flow behavior towards zero stresses (complete liquefaction). At the beginning of shearing, the assembly exhibits strain hardening and reaches an initial peak at very small shear strain i.e. γ is around 1%. The initial peak is referred to as the triggering of liquefaction in CS theory. After the triggering point, the strain-softening behaviour takes place until reaching zero stresses at CS i.e. complete liquefaction.

3.3 *Critical state lines, CSLs*

Fig. 4a shows a unique CSL in the q-p' space for both simple shear and triaxial simulations. Note, the triaxial simulation results can be found in Nguyen et al. (2017). Therefore, a unique M value, which directly relates to the CS friction angle or angle of repose for a soil, can be observed for both types of simulations. However, the CSLs in e-log(p') space were lightly different. Note the non-uniqueness of the CSL was reported in the literature due to the difference in the stress Lode angle.

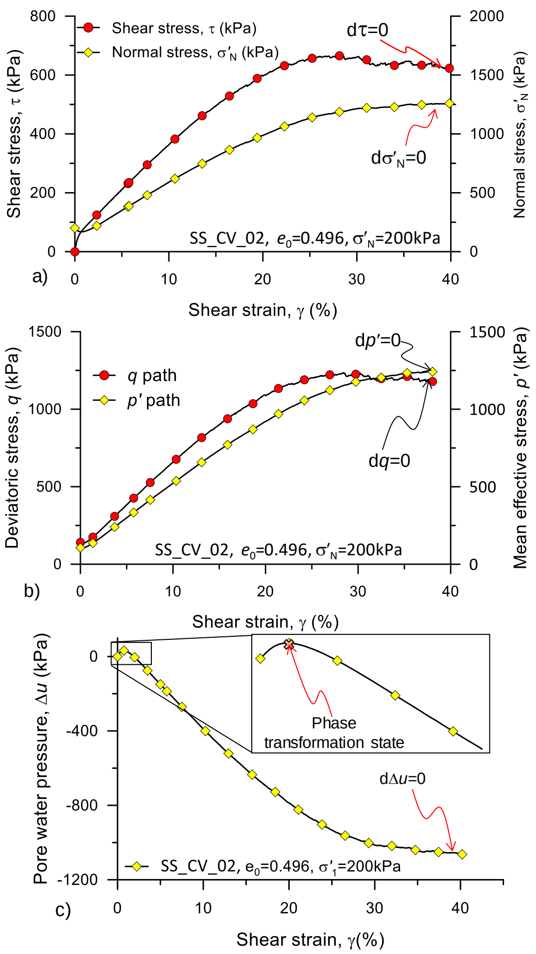

Figure 2. Constant volume behaviour under simple shear condition for non-flow behavior: (a) τ-σ′N-γ, (b) *q*-*p*′-γ and (c) Δ*u*-γ spaces.

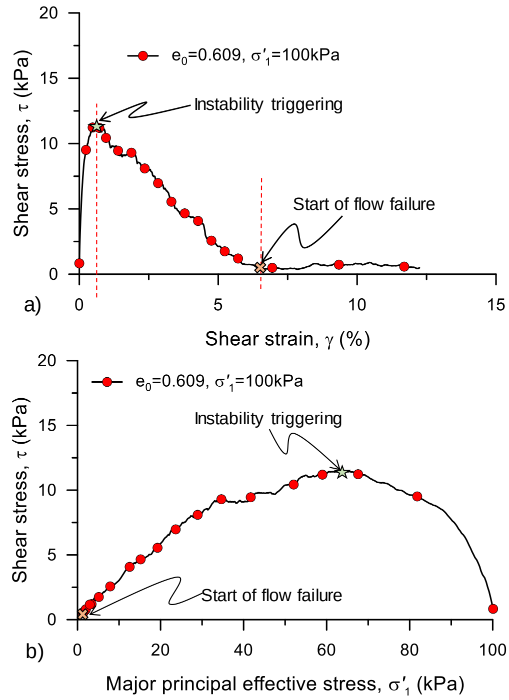

Figure 3. Constant volume behaviour under simple shear condition for flow behavior: (a) τ-γ and (b) τ-σ′N spaces.

The CSL in *e*-log(*p*′) space of simple shear simulations is lower than that of triaxial simulations, as shown in Fig. 4b. The general formulation of the classical CSL in *e*-log(*p*′) space can be written as:

$$e = e_{lim} - \lambda\left(p'/p_{ref}\right)^{\xi} \tag{2}$$

where p_{ref} is reference stress of 101kPa; e_{lim}, λ and ξ are CS parameters. As mentioned previously, in laboratory simple shear tests, *p*′ may not be known, which makes it challenging to confirm the results with the classical CS theory. Therefore, Fig. 4b would help to investigate the different CSL for simple shear condition and modify the CSSM framework for such condition. In addition, Fig. 4c shows that the stress ratio (τ/σ′N) at CS is unique, but this stress ratio is comparatively smaller than *M* in triaxial conditions.

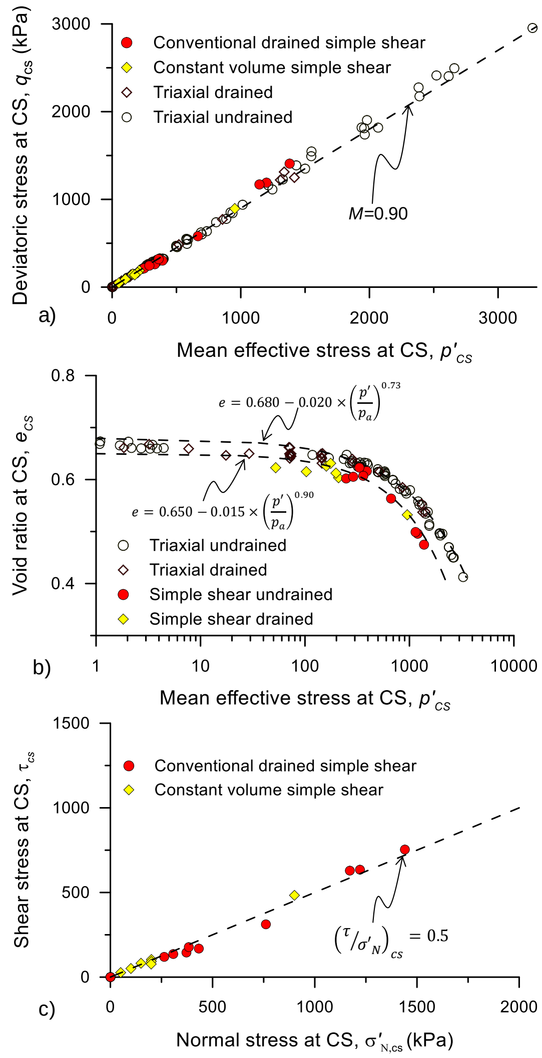

Figure 4. CSL for both simple shear and triaxial conditions in (a) *q*-*p*′, (b) *e*-log(*p*′) and (c) τ-σ′N (simple shear only) spaces.

4 CONCLUSIONS

This study aims to modify the CS theory for the simple shear test by investigating the equivalence between the simple shear and triaxial conditions. The study takes advantage of using DEM, which can measure all stress and strain components in both principle and shear directions, for the investigation. The main findings of this study include:

- In simple shear condition, all stress and strain components (i.e. τ, σ'_N and ε_{11}) evolve towards the CS with the increasing shear strain (γ). The CS in simple shear condition then should be defined as $d\tau=d\sigma'_N=d\varepsilon_{11}=0$, whereas the CS in triaxial condition is defined as $dq=dp'=d\Delta u=d\varepsilon_v=0$.
- The simple shear simulations also showed the contractive and dilative tendencies, which is in line with the triaxial test in the literature. Say, the dense specimen exhibited strain hardening as γ increased. The loose specimen showed flow liquefaction after reaching an initial peak τ.
- It was found that the CSL of the simple shear simulation was lower than that of triaxial simulation in the classical *e*-log(*p*′) space. However, the *M* value in the *q*-*p*′ space, which directly relates to the critical state friction angle, is approximately the same for both conditions.

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